THE KING'S SCHOOL, CANTERBURY

SCHOLARSHIP ENTRANCE EXAMINATION

February 2012

MATHEMATICS 2

Time: 60 minutes (plus reading time)

Use the reading time wisely; gain an overview of the paper and start to think of how you will answer the questions.

Do as many questions as you can (clearly numbered) on the lined paper provided. Clearly name each sheet used. You are encouraged to attempt these questions in order.

The questions are not of equal length or mark allocation. Make sure you avoid spending too much time on any one question; don't get bogged down! Move on quickly if you get stuck. The paper is quite long; you are not necessarily expected to finish everything.

Some of the later questions are more difficult, but not necessarily longer. Some questions are designed to test your ability to work with unfamiliar ideas, or familiar ones with a twist. Don't give up!

You are expected to use a calculator where appropriate, but also you must show ***full and clear working****, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers. In fact, merely writing down an answer might score very few marks.*

Complete questions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.

This paper has eleven questions.

1 (a) Write down the value of $\sqrt{2012}$ to 3 s.f.

(b) Showing your reasoning or working very carefully, find how many positive values of *x* make the following expression equal a whole number (**you do not need to work out the values of *x***)?

$$\sqrt{2012 - \sqrt{x}}$$

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2 Mr Zee and Mr West sit a long scholarship examination which has 99 problems. Each problem has ten marks available.

Mr Zee scores an average of 70% on the first 90 problems and after that he only scores 10% on the final 9 problems.

Mr West scores an average of 90% on the first 9 problems and then scores 60% on the final 90 problems.

(a) Without making any calculations, who do you think has scored better overall. Why?
[n.b. you cannot get this part of the question wrong if you answer it.]

(b) Now work out carefully how each of the candidates did overall, and draw a conclusion based on these calculations.

(c) Comment on your conclusion; how does this compare with what you said in (a)?

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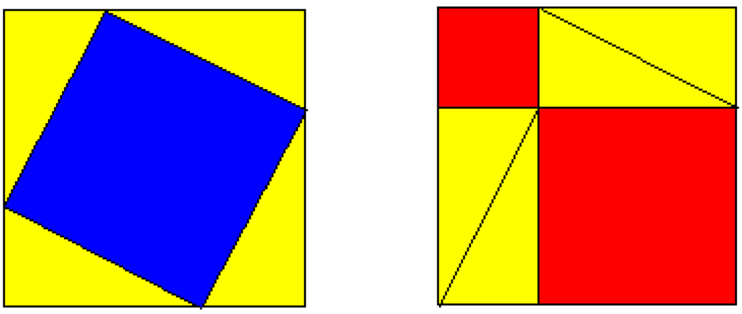
3 Leonardo of Pisa, also known as Fibonacci, published his *Liber Abbaci* in 1202. This book was a collection of mathematical problems, including the one below, called “On four pieces of cloth”.

*A certain man bought 4 pieces of cloth for 154 bezants [an old money currency], the first of which he bought for **something**, and the second he bought for* $\frac{2}{3}$ *the price of the first. Moreover the third he bought for* $\frac{3}{4}$ *the price of the second, and the fourth he bought for* $\frac{4}{5}$ *the price of the third.*
It is sought how much each piece was worth.

Your task in this question is to find out how much he paid for each piece of cloth.

[Hint: call the price (“something”) of the original piece *x* , say, and then set up and solve an equation in *x*.]

4



Suppose the identical right-angled triangles in the diagrams above have sides of length *a*, *b* and *c*, where *c* is the **hypotenuse** (longest side).

(a) Can you explain why each of the diagrams shows a large square of equal size?

(b) Without using any real algebra, can you **explain** how these diagrams demonstrate the truth of the Pythagorean Theorem, stated below?

$$a^2 + b^2 = c^2$$

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5 At the Casino Royale, James Bond orders a martini cocktail and it arrives in a conical glass, like this:

The formula for the volume of a cone is:

$$V = \frac{1}{3}\pi r^2 h$$

where *r* is the radius of the cone and *h* its height.

Bond's glass (**conical part only**) has height 10 cm and a radius of 5 cm at the top.

The liquid inside has a depth of 7 cm. The glass is very thin (i.e. we ignore its thickness).

(a) How full is the glass (give your answer as a percentage or a fraction)?
[Hint: the volume of liquid may also be seen as a cone]

(b) Comment on the size of your answer.

(c) Showing your working carefully, work out what depth of liquid would half fill the glass.

A martini is made with two ingredients: vodka and vermouth. This one has them in the ratio 4:1 respectively.

Bond prefers them in the ratio 5:1 instead, so orders his next drink in this proportion. Both drinks are of identical size.

(d) Work out the percentage decrease in vermouth in moving from the first to the second drink.

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6 (a) Uncle Monty roasts some meat in the oven. He adds garlic, rosemary, and salt in random order.

Explain carefully, **without writing all the possibilities down**, why there are six different ways in which he can add these three ingredients to the meat (i.e. try and show the calculation).

(b) The following shows a scholarship examination room with eight desks.

1	2	3	4
5	6	7	8

There are six candidates and two teachers. Desk 1 must be occupied by a teacher (it can be either).
The other teacher sits at one of the desks which is not next to desk 1 (so, not 2, 5 or 6).

Explain very carefully why there are 5760 possible arrangements of the people in the room.

7 The UK Gas Mark (G) on gas cookers is related by a formula to the temperature using the older scale of degrees Fahrenheit (°F).

(a) Using the table below, work out an algebraic formula for F in terms of G.

Gas Mark G	Temperature in °F
1	275°
2	300°
3	325°
4	350°
5	375°
6	400°
7	425°
8	450°
9	475°

Uncle Monty is still trying to cook the joint of meat from Question 6 and his recipe says the oven should be heated to Gas Mark 5. Unfortunately, he cannot read the markings on the oven clearly, so he turns the oven on and guesses the heat setting.

He tests the oven temperature with his thermometer but it is only marked in degrees Celsius (°C). His thermometer shows the oven temperature is 170 °C. Luckily, he knows a formula to convert between Celsius and Fahrenheit. It is

$$F = \frac{9}{5}C + 32$$

(b) Showing your working clearly, explain whether he is over- or undercooking the meat, or whether it is just about right.

8 Consider some numbers:

$$1^5 = 1$$
$$2^5 = 32$$
$$3^5 = 243$$

(a) Continue this pattern, writing out the fifth powers, for the next three lines.

(b) What do you notice about the last digit of your answer each time?

(c) From (b), can you make a guess as to what is **always** true about $a^5 - a$ (where a is a positive whole number)?

To try and prove this we will assume the truth of **Fermat's Little Theorem**, which states:

> If a is a whole number and p is a **prime** number then
>
> $a^p - a$ is divisible by p i.e. p is a factor of $a^p - a$

(d) What does Fermat's Little Theorem tell us about $a^5 - a$?

(e) Moreover, can you explain why $a^5 - a$ is always even?

[Hint: you might like to consider separate cases when a is odd or even].

(f) What can you conclude from parts (d) and (e) that we can say is always true about $a^5 - a$?

9 In the near future there are 100 television channels, each of which is broadcasting a programme. All 100 programmes have some or all of the following components in their title **(do not worry about their order)**:

- Cats
- Dancing
- On Ice
- Have Talent

There is no other type of TV programme.

(a) Fill in the Venn diagram on the separate sheet using the following information, and then answer the questions below.

i. There is only one show called ***Cats Dancing on Ice Have Talent***.

ii. There are X shows each with only **two** components i.e. X of ***Cats on Ice***, X of ***Dancing has Talent***, etc.

[Hint: you should find six of these regions]

iii. There are six shows just about ***Cats***, seven shows just about ***Dancing***, and eight shows only ***On Ice***.

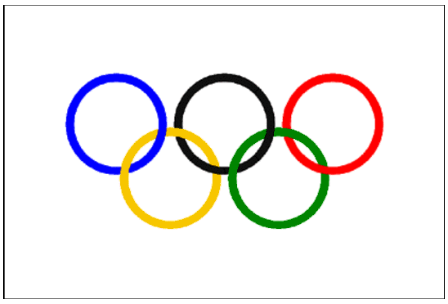
Other totals are already marked on the diagram.

(b) How many shows just called ***Have Talent*** are there?

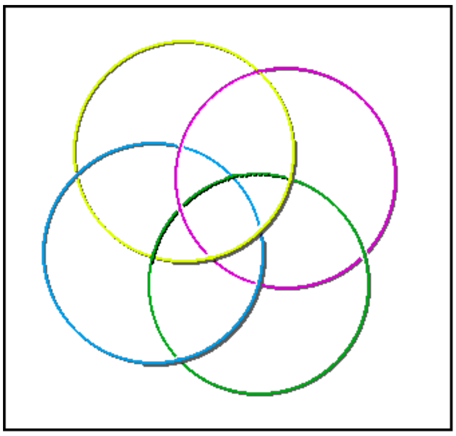
(c) How many shows called ***Cats Dancing On Ice*** are there?

(d) Form and solve an equation in X to find its value.

(e) Next, can you explain clearly why the diagram below is probably not useful as a Venn diagram for **five** sets?



(f) Can you explain clearly why the diagram below is probably not useful as a Venn diagram for **four** sets?



10 Alex and Jordan decide to try and complete a scholarship paper.

Alone, Alex can complete it in three hours without a break.
On his own, Jordan can finish it in five hours without stopping.

Instead, they work through the paper together, taking a one-hour coffee break at some point.

Suppose t is the total time (in hours, including the break) needed for them to complete the paper together.

Henson, Lewis and Jonathan try and find out how long it will take Alex and Jordan to finish the paper.

They each come up with an equation, the solution to which they claim will answer the question.

Only one of their equations is correct.

Henson thinks the equation to solve is $(5+3)t = 1$

Lewis thinks the equation to solve is $\left(\frac{1}{3}+\frac{1}{5}\right)(t-1) = 1$

Jonathan thinks the equation to solve is $\left(\frac{1}{3}+\frac{1}{5}\right)(t+1) = 1$

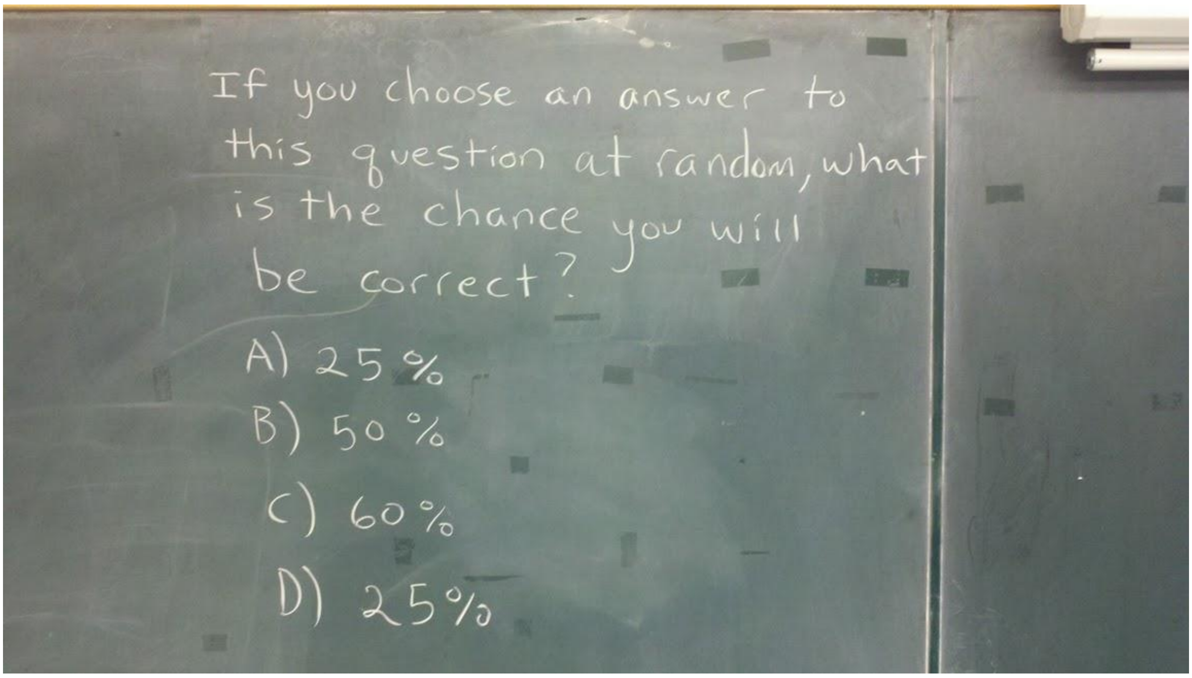
(a) Showing your reasoning carefully, say which of these equations is the correct one, and why.

(b) Solve this equation to find how many hours they take to finish the paper together.

[Hint: it may be helpful to think about the rates at which they each work through the paper].

11 *You should try and make sure you have done as much as you can of the other questions before attempting this bonus problem.*

Write a few sentences and/or calculations to try and explain how you might answer this question, if it can be answered at all. Imagine you are attempting to explain this question to someone else; try and say something intelligent!



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Scholarship mathematics 2012
Extra sheet for Question 9

Write your name here: ______________________________